Transcendental Syntax

The dynamics of logic programs and tilings, applied to Linear Logic

Team LoVe – LIPN Université Sorbone Paris Nord

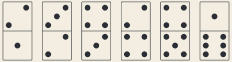
Boris ENG

Stellar Resolution

From tiles to logic programs

Wang tiles

Hao Wang (1961)

Dominos. 

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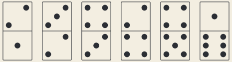
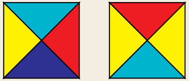
Dominos. 

Wang set. 

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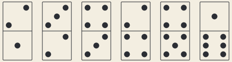
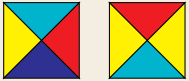
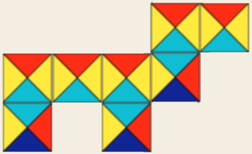
Dominos. 

Wang set. 



Tiling. $\alpha : \mathbf{Z}^2 \longrightarrow T$, adjacent tiles : sides of matching colours.

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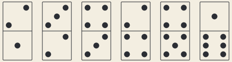
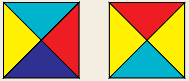
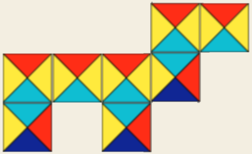
Dominos. 

Wang set. 

Tiling.  $\alpha : \mathbf{Z}^2 \longrightarrow T$, adjacent tiles : sides of matching colours.

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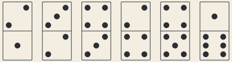
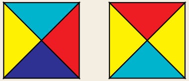
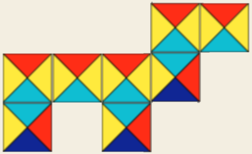
Dominos. 

Wang set. 

Tiling.  $\alpha : \mathbf{Z}^2 \longrightarrow T$, adjacent tiles : sides of matching colours.

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Generalisations. different matchability (e.g DNA computing), higher dimensions (e.g $\mathbf{Z}^3$).

Flexible tiles

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Coming from DNA computing.

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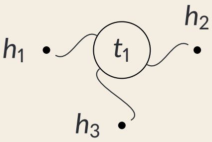

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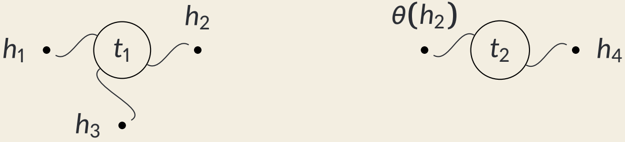

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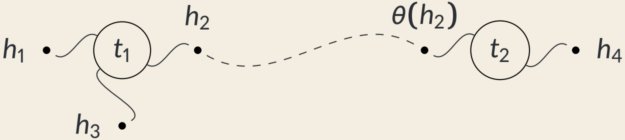

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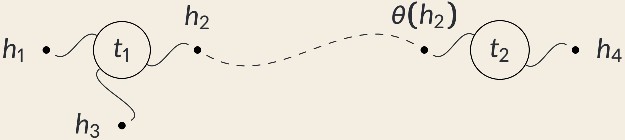

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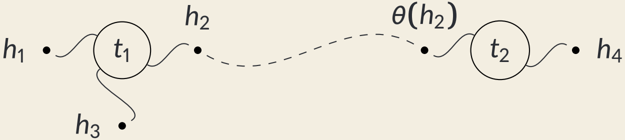

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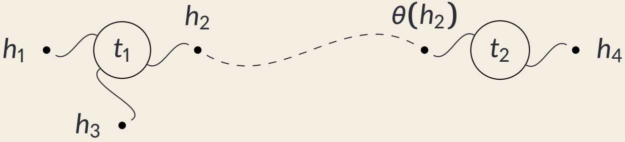

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- Related to `NTIME` classes.

  "*Complexity classes for self-assembling flexible tiles*" (Jonoska et al.).

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Flexible tiles on first-order terms.

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Unification. $t_1 \doteq t_2$: can we find $\theta : Vars \longmapsto Terms$ such that $\theta t_1 = \theta t_2$?

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↳ for $x \doteq f(x) \simeq_\alpha y \doteq f(x)$ we have $\theta = y \mapsto f(x)$

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Stars and constellations

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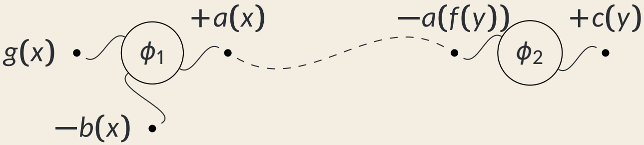

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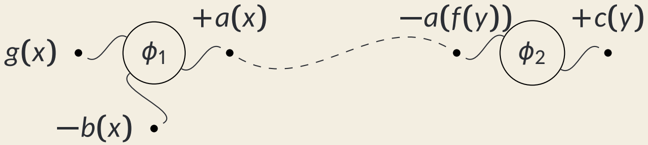

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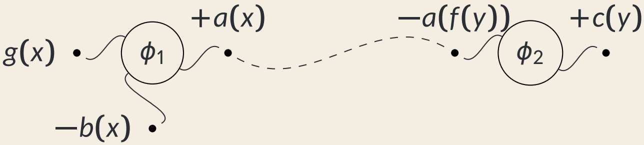

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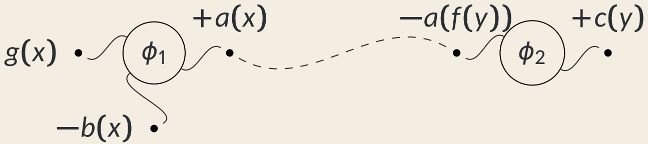

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Tiles	Resolution	Stellar Resolution
	Atom $A = A(t), \neg A(t)$	Ray $r = +a(t), -a(t), t$
Tile	Clause $C = A_1 \vee \ldots \vee A_n$	Star $\phi = [r_1, \ldots, r_n]$
Tile set	Program $P = C_1 \wedge \ldots \wedge C_m$	Constellation $\Phi = \phi_1 + \ldots + \phi_m$
Tiling	Inference tree	Diagram

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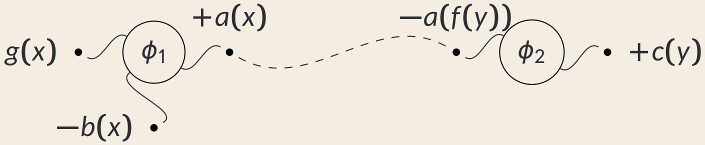

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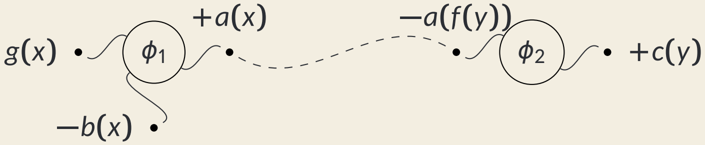

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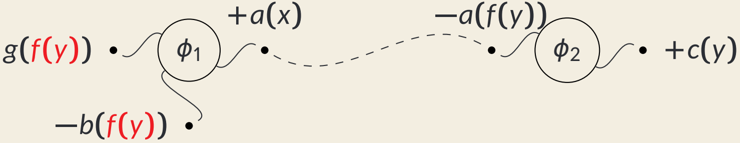

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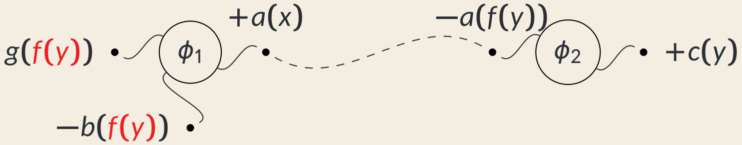

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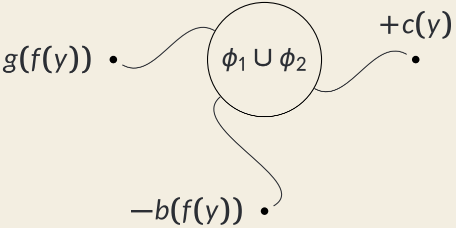

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ϕ_3

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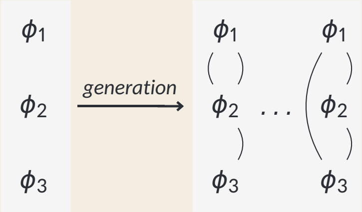

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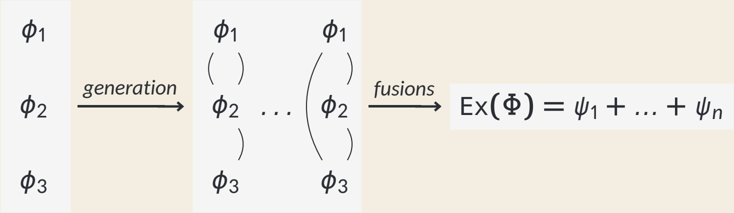

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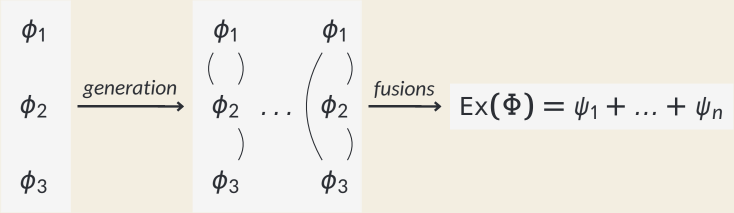

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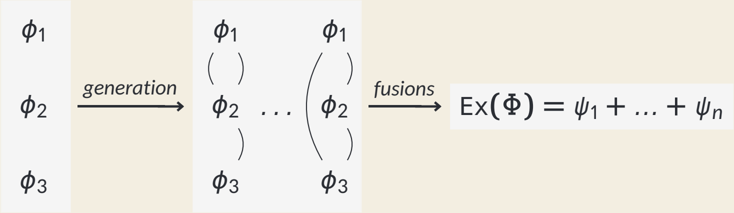

- We want the diagrams to be saturated (impossible to extend).
- We also want them to be correct (no unification error).

Stellar Resolution

Few examples

Unary addition by **logic programming** :

$$[+add(0,y,y)] + [-add(x,y,z), +add(s(x),y,s(z))] + [-add(s^n(0),s^m(0),r),r]$$

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All other diagrams fail, hence $\mathrm{Ex}(\Phi) = [s^4(0)]$.

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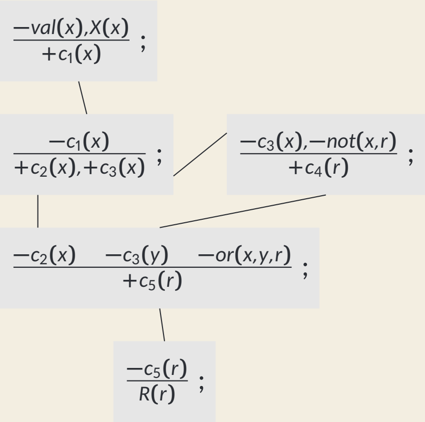

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Extensible to arithmetic circuits

Transcendental Syntax

Proofs as tilings

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Explain (linear) logic from its computational behaviour.

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Answers/Analytic adequate computational space : stellar resolution.

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"*This can only be a reconstruction, which means that we roughly know what we are aiming at.*" (Transcendental Syntax I).

Transcendental Syntax

MLL proof-structures

An alternative representation of proofs as hypergraphs :

Transcendental Syntax

MLL proof-structures

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Formulas/Vertices. $A, B := X \mid A \otimes B \mid A \parr B$

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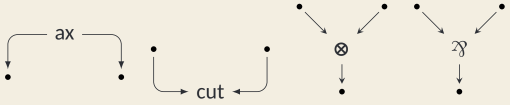

Rules/Hyperedges. Axiom Cut Tensor Par

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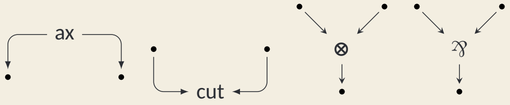

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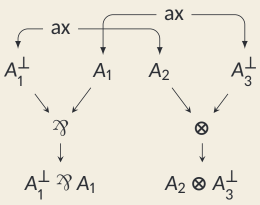

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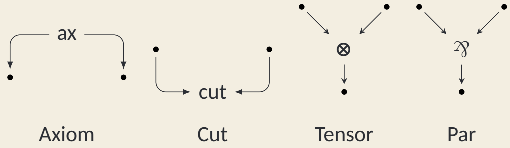

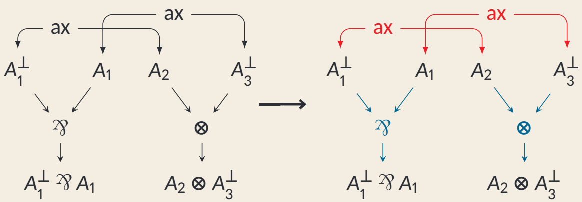

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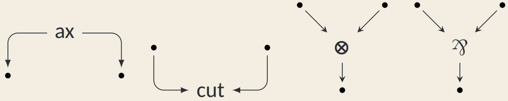

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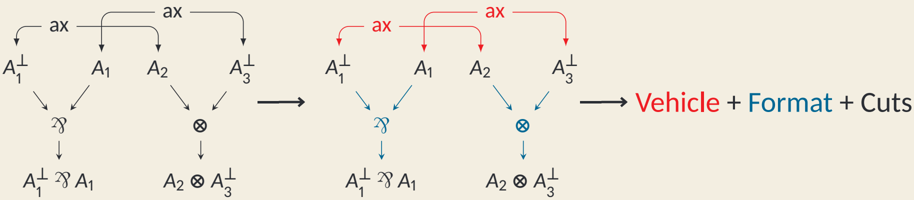

Transcendental Syntax

The computational content of proof-structures

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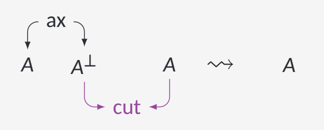

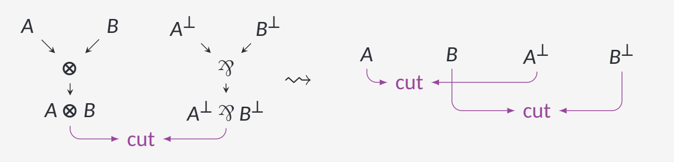

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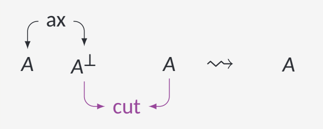

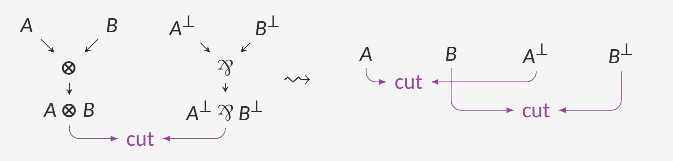

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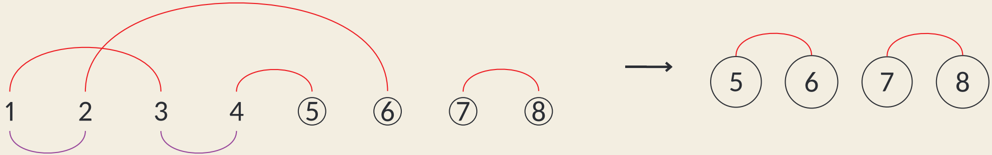

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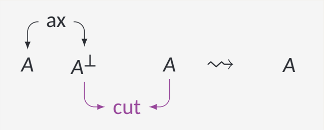

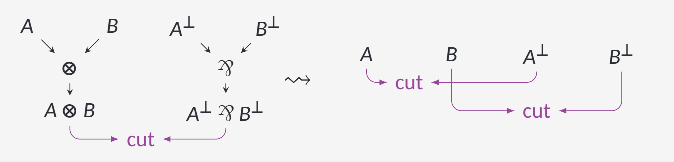

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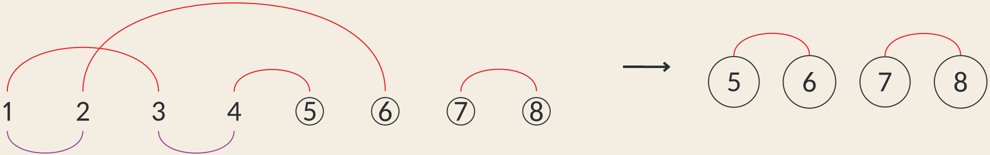
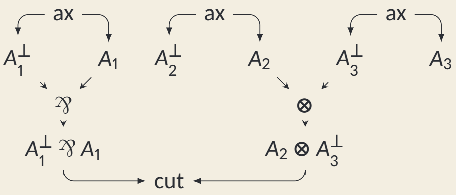
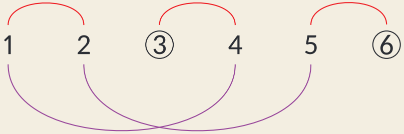


Basically a computation of maximal paths in a graph.

Transcendental Syntax

Simulation of cut-elimination





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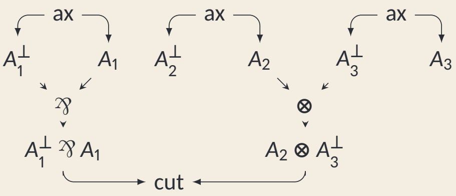

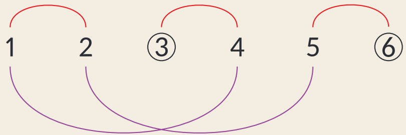

$+c.p_{A_1^\perp ⅋ A_1}(\mathtt{l}x);\ +c.p_{A_1^\perp ⅋ A_1}(\mathtt{r}x);$

$+c.p_{A_2^\perp}(x);\ +c.p_{A_2 \otimes A_3^\perp}(\mathtt{l}x);$

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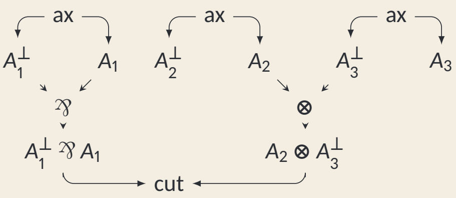

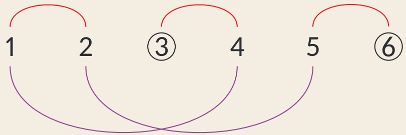

$+c.p_{A_1^\perp \parr A_1}(\mathtt{l}x);\quad +c.p_{A_1^\perp \parr A_1}(\mathtt{r}x)\ ;$

$+c.p_{A_2^\perp}(x);\quad +c.p_{A_2 \otimes A_3^\perp}(\mathtt{l}x)\ ;$

$+c.p_{A_2 \otimes A_3^\perp}(\mathtt{r}x);\quad +c.p_{A_3}(x)\ ;$

$-c.p_{A_1^\perp \parr A_1}(x);\quad -c.p_{A_2 \otimes A_3^\perp}(x)\ ;$

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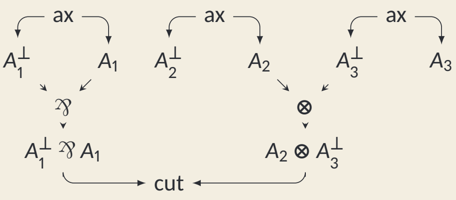

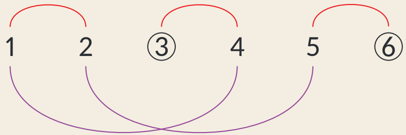

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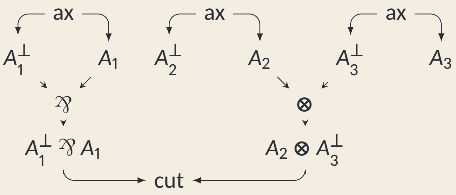

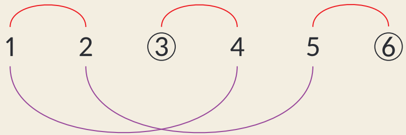

$+c.p_{A_2^\perp}(x)$; $+c.p_{A_3}(x)$;

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The logical content of proof-structures

Only some proof-structures are "logically correct".

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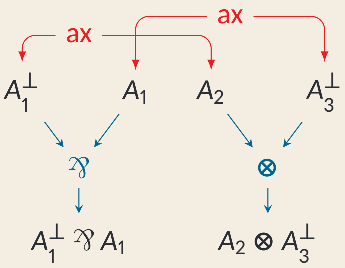

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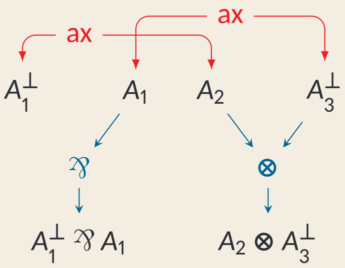

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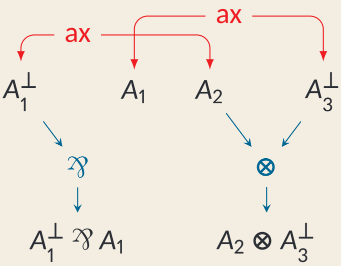

Transcendental Syntax

Simulation of correctness

Stars $\equiv$ Oriented hyperedges

$+t.p_{A\otimes B}(\mathtt{l}x);\ \ +t.p_{A^\perp \parr B^\perp}(\mathtt{l}x)\ ;$

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$+t.p_{A\otimes B}(lx);\ \ +t.p_{A^\perp ⅋ B^\perp}(lx)\ ;$

$+t.p_{A\otimes B}(rx);\ \ +t.p_{A^\perp ⅋ B^\perp}(rx)\ ;$

$[\frac{-t.p_{A\otimes B}(lx)}{+c.q_A(x)}]\ ;$

$[\frac{-t.p_{A\otimes B}(rx)}{+c.q_{A^\perp}(x)}]\ ;$

$[\frac{-t.p_{A^\perp ⅋ B^\perp}(lx)}{+c.q_B(x)}]\ ;$

$[\frac{-t.p_{A^\perp ⅋ B^\perp}(rx)}{+c.q_{B^\perp}(x)}]\ ;$

$[\frac{-c.q_A(x)\quad -c.q_B(x)}{+c.q_{A\otimes B}(x)}]\ ;$

$\frac{-c.q_{A^\perp}(x)}{}\ ;$

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$[\frac{-c.q_{A\otimes B}(x)}{p_{A\otimes B}(x)}]\ ;$

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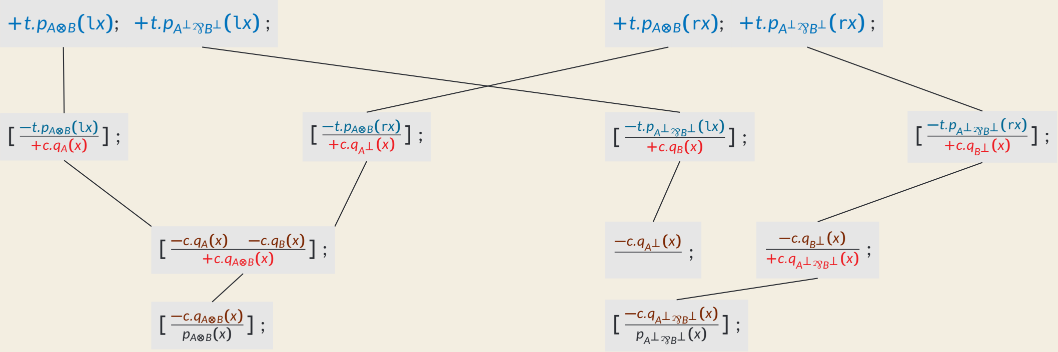

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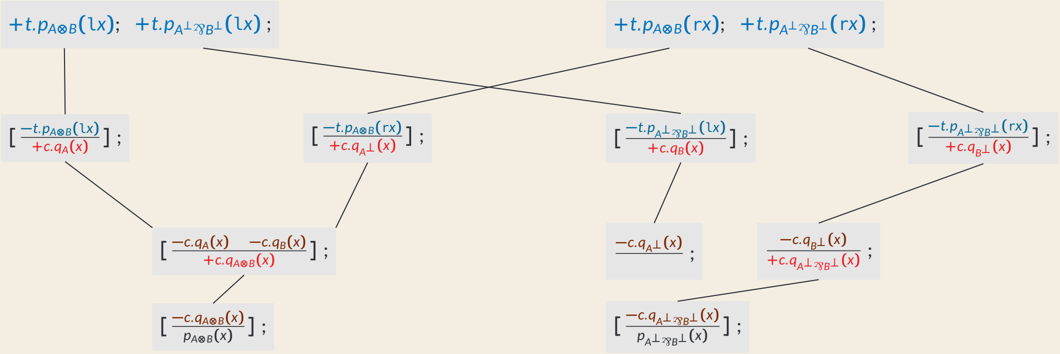

Property of the tiling : **logically correct** iff for all test, the normal form is a single star.

Transcendental Syntax

Typing and formulas

Use of techniques from "linear" realisability.

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$\Phi_1 \perp \Phi_2 \iff |Ex(\Phi_1 \uplus \Phi_2)| = 1$: captures MLL formulas.

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  ↳ terms/individuals as multiplicative propositions
  ↳ equality as linear equivalence (not predicate !)
- Logic programs and functional programs, unified ?

Future and related works

(actually a call for help)

Hypergraphings

Hypergraphs with dynamics

Computation with hypergraphs :

Model	**Vertices**	**Hyperedges**
Boolean circuits	addresses	gates
Proof-nets	addresses	rules
Constellations	terms	stars
Automata	states	transitions

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↳ generalisation of Seiller's *Graphings*.

↳ categorical framework ? Operads, string diagrams, hypergraph categories, frobenius algebras, ...

Stellar Resolution and Automata Theory

Dependency graph $\mathfrak{D}(\Phi)$: relations of matchability within a constellation Φ.

$$\begin{tikzcd} -add(0,y,y); \arrow[r, dash] & +add(x,y,z);\ \ -add(s(x),y,s(z)); \arrow[loop above, dash] \arrow[r, dash] & +add(s^n(0),s^m(0),r);\ \ r; \end{tikzcd}$$

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Diagram (formally) : graph homomorphism $\delta : G \to \mathfrak{D}(\Phi)$.

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Run in finite automata : path $\longmapsto$ state graph

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↳ we are interested in reaching finale state.

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Diagram (formally) : graph homomorphism $\delta : G \to \mathfrak{D}(\Phi)$.

$-add(0, y, y);$ — $+add(x, y, z);\ \ -add(s(x), y, s(z));$

$+add(x, y, z);\ \ -add(s(x), y, s(z));$ — $+add(s^2(0), s^2(0), r);\ \ r;$

Run in finite automata : path $\longmapsto$ state graph

↳ we are interested in reaching finale state.

↳ automaton for stellar execution ?